

**Differentiation**

Review year 12

Differentiation Rules

1. If then, where and are numbers.
2. Derivative of a constant term (number) is zero.
3. A constant factor is not affected by the process of differentiation.
4. A polynomial function is differentiated term by term.

Note

1. Brackets must first be expanded and then differentiate.
2. Where possible split the fraction into terms and then differentiate.

Examples: Differentiate the following.



Notations:

First derivative

.

(Means derivative of y with respect to x)

Second derivative

**Definition of Derived Function (first principles’)**

Given a function , we wish to find the gradient of a line joining points A and B.

The gradient of the straight line connecting points A and B equals

This gradient is an approximation to the gradient of the curve at A. The smaller , the closer the two points and the better the approximation.

This leads to the definition of derived function:

**Note:** The denominator of usually cancels out with part of the numerator when the numerator is simplified and factorised.

Examples:

Differentiate these functions using first principles.

1. \*

**Chain Rule** or Composite Functions or Function of a Function Rule

A composite function is a function nested in another function. For instance is a composite function then

**OR**

We can separate the two function and write them as

If

Note: Chain rule is used for functions with brackets.

Example: Differentiate the following;

**Trigonometric Functions**

Differentiation rules for trig functions:

|  |  |
| --- | --- |
| Rules | Short chain rules |
|  |  |
|  |  |
|  |  |

Where and

Derivative of Reciprocal trig functions

Examples: Differentiate the following;

**Exponential and Logarithmic Functions**

Exponential

Only the function has this unique property; its derivative is the function itself.

If

Using chain rule:

If

**Logarithm** (ln)

If note

Using chain rule;

If

Note:

Functions may need to be simplified first, using log rules.

Example

1. Differentiate the following:

B. Differentiate the following

**Chain Rule (Extension)**

Differentiate. No need to simplify your answers.

**Product Rule**

When a function is the product of two simpler functions we use the product rule for differentiation:

If

In short or

Example: Differentiate the following;

**Quotient Rule**

When a function is the quotient (fraction) of two simpler functions, we use quotient rule for differentiation.

If then

In short

Example: Differentiate the following:

**Note**: Do not simplify means; show the rules in your differentiation of a function

**Recap**

Differentiate. No need to simplify your answers.

**Implicit Relation**

There are two types of relation, explicit and implicit. An explicit relation is the one which can be solved, whereas an implicit relation is the one which cannot be solved (cannot write one variable as a subject).

e.g.

**Steps to differentiate an implicit relation are:**

1. *Differentiate each term with respect to*
2. *Use chain or product or quotient rule to differentiate each term.*
3. *Rearrange and write as the subject.*

Example: differentiate the following.

**Second derivatives (**

Second derivative simply means you differentiate the first derivative function using chain, product or quotient rules or combinations.

**Examples**

1. For the given function, find
2. For the function show that

1. Given , show that

**Parametric Relation**

It is a relation which has three variables, and an introduced variable.

The variable is called a **parameter** of the equation. Eliminating from the parametric equations gives the **Cartesian** equation.

To differentiate a parametric equation use **chain rule**:

where is the parameter.



Note:

**Second derivatives (**

It simply means you differentiate the first derivative function in terms of parameter () and multiply it by

**For second derivative use chain rule see example 2**

Example 1: If and

1. Find the Cartesian equation.
2. Find in terms of

Example 2: If and

Find

Example 3: A cycloid has parametric equations

. Show that

Example 4: Find the second derivative for

.

